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Learning from People, Things, and Signs

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LEARNING FROM PEOPLE, THINGS, AND SIGNS

ABSTRACT. Starting from the observation that small children can count more objects than numbers—a phenomenon that I am calling the "lifeworld dependency of cognition"—and an analysis of finger calculation, the paper shows how learning can be explained as the development of cognitive systems. Parts of those systems are not only an individual's different forms of knowledge and cognitive abilities, but also other people, things, and signs. The paper argues that cognitive systems are first of all semiotic systems since they are dependent on signs and representations as mediators. The two main questions discussed here are how the external world constrains and promotes the development of cognitive abilities, and how we can move from cognitive abilities that are necessarily connected with concrete situations to abstract knowledge.

KEY WORDS: Lifeworld dependency of cognition, implicit knowledge, distributed and situated cognition, cognitive apprenticeship, scaffolding, internalization, shared intentionality, semiotics, diagrammatic reasoning, pragmatism, Peirce, Vygotsky

If you have a four or five year old child, you can make an astonishing observation. When you ask the kid to count as far as it can, it might come to 6 before it gets difficult. But ask it to count wood blocks and it might come without any difficulties to 26 (cf. Hasemann, 2003, pp.5–6; Caluori, 2004, pp.153, 252ff.). The disturbing question is: Does the child *know* the numbers from 1 to 26, or does it not? If it knows the numbers, why, then, is it not able to count them without having the wood blocks in front of it? And if it does not know these numbers, it should be impossible to count anything.

The problem that becomes visible here is obviously the question of what we mean by "knowing" something. In psychology, a distinction between "implicit" and "explicit" knowledge has been suggested to describe observations like the one mentioned above. Arthur Reber (1989) showed that what we know "implicitly" is always "ahead of explicit knowledge" (p. 229). In empirical studies he performed, subjects had to learn the syntax of an artificial grammar. It turned out that even if subjects improved their abilities to *explicate* the knowledge they generated while learning, their *implicit* knowledge of this syntax was always richer. The "former never caught up with the latter; that is, as subjects improved in their ability to verbalize the rules that they were using, they also developed richer and more complex rules. Implicit knowledge remained ahead of explicit knowledge" (p. 229).

However, the essential point with the counting child is the activity with concrete objects. It is in *doing* something that we develop just those abilities we need to cope with the objects of our activity. Since what Reber calls "implicit knowledge" seems to be dependent on concrete activities with things, it might be more appropriate to talk about "cognitive systems" than about "implicit knowledge." While the latter is clearly associated with something that can be located in an individual's mind, the former concept is linked to a recent discussion in cognitive science that focuses on "distributed," "situated," or "embodied cognition" (Hutchins, 1995; Clark, 1998; cf. also the idea of an "extended mind" in Clark & Chalmers, 1998). Basic for these approaches is the idea that an individual's cognitive abilities can only be understood as parts of "cognitive systems" that include the respective environment and social settings as "driving forces" for cognitive processes. But if cognition depends on the respective environment, it seems to be clear that in different environments—counting numbers *versus* counting objects—different cognitive abilities are involved.

These differences hint at something very mysterious. How are our cognitive abilities connected with their respective *Umwelt*? How is it possible to cope cognitively with the fact that those environments are highly complex? They do not only contain a lot of things, but also different people. An interview situation is first of all a *social* situation. Most mysterious is, however, how all this affects the possibility of *learning*.

To answer these questions, the first goal of this paper is to clarify the concept of a "cognitive system." Based on a more detailed analysis of the counting example on the one hand, and a further example in which a child solves a problem by calculating on her fingers on the other, I suggest a distinction between "knowledge" and "cognitive abilities." While knowledge is "at our disposal," abstracted from environmental factors, cognitive abilities are dependent on something else—as the "ability" to count 26 wood blocks is dependent on the existence of those blocks. Since the observations of both Hasemann and Reber show that what I am calling cognitive abilities *precedes* knowledge, I will focus in the first sections of this paper on two questions: First, how does the child's activity—either with the wood blocks, or with her fingers as a representational system—promote the development of her cognitive abilities? And second, how is it possible to develop knowledge based on cognitive abilities?

The second goal is to elaborate and defend the thesis that all cognitive systems are semiotic systems, that is systems whose relations are mediated, first of all, by signs and representations. That opens up a new perspective for cognitive science: to reconstruct cognition by means of semiotic theories. Based on what we know about the problems of interpreting signs, this will lead to a new formulation of problems we have to face regarding the cognitive processes involved in learning.

COGNITIVE SYSTEMS: KNOWLEDGE *VERSUS* COGNITIVE ABILITIES

Since the concept of a "cognitive system" seems to be most appropriate to understand the two different results of the counting child, we must first clarify the meaning of this concept. A *cognitive system*, according to the definition I would suggest, is constituted by a set of elements (called "objects" here, or the system's "ontology") on the one hand, and a set of relations between those elements on the other. It is a dynamic system. That means, the objects and relations can only be specified with regard to a certain moment and situation, and this only hypothetically. A cognitive system in a certain situation includes all those objects and relations that are relevant to explain the possibility of cognitive activities that happen in this situation. Cognitive activities are those internal (i.e. mental, to describe by neurophysiological means) and internal-external (social or object related) processes we have to presuppose to explain the possibility of any activity we are able to perform (walking, eating, perceiving, memorizing, understanding, explaining, problem solving, sign usage, communication, collaboration, defending, attacking, taking care, fulfilling responsibilities, and so on).

While it is easy to formulate such a formal definition of a cognitive system, it is hard to specify those systems for concrete cases. However, if we look at the child that counts 26 wood blocks but only 6 numbers we can be a bit more specific—and we can get some insights why it is difficult to specify concrete, cognitive systems. In order to grasp the difference between counting wood blocks and counting numbers I suggest the terminological distinction between a *cognitive ability* in the first case, and *knowledge* in the second. I reserve the term "knowledge" for what is *abstracted* from those concrete activities in which we might use this knowledge. While we have "knowledge" *at our disposal*—"ready-made," so to speak—our "cognitive abilities" depend on, and are determined by, concrete activities in a way that when those activities, or the involved objects, are changing, also this ability is changing. Knowledge can either be *explicit* or *implicit*, and in both cases it can either be given in *propositional* or *non-propositional* form. "Propositional" knowledge is "explicit" in the very moment when it is explicated in a form like "I know that ..." (cf. Ryle, 1949), and it is "implicit" in case somebody is able to formulate such a proposition. "Non-propositional" or "practical" knowledge corresponds to Ryle's "knowing how," and it can either be explicit (observable in a concrete situation) or implicit, for example if somebody is able to perform an activity based on what could be reconstructed as a set of instructions.

It is decisive for propositional and non-propositional forms of *implicit knowledge* that they are at our disposal independently of concrete activities or objects. This way, a software code would be a form of "non-propositional implicit knowledge" since what the software is doing can be explicated completely in the form of instructions, and these instructions do not change in dependence of the program that is executed. But if a child is playing with wood blocks, we would observe a "non-propositional *cognitive ability*" since we do not know whether the child's ability to do these things depends on the presence of these wood blocks, and would change if it would play with other objects.

The point is, a cognitive ability can be described by the *observable function* it fulfils in a *concrete* lifeworld situation while implicit knowledge can be expressed in the form of sentences that refer to *intended applications* or *possible functions*. That means, if a child is able to count the numbers from 1 to 6, I would say it has the *implicit knowledge* for doing so *in general*, but if it is able to count up to 26 wood blocks, then it has the *cognitive ability* to do this in this specific lifeworld situation.

While it seems to be clear in the first case that the ability to count the numbers does not depend on a concrete activity, it is impossible in the second case to describe what exactly the kid "knows." It is impossible because in this case the cognitive system of which this ability is a part cannot be completely described because we do not know which parts of the external environment are essential parts of this system and which are not. We do not know whether the same child would be able to count a mixed set of wood blocks and Lego bricks, or 13 red and 13 white blocks, or wood blocks set in a single line or in rows of five, or in a straight or curved line, and so on. Being dependent on a cognitive system which includes elements of the environment, we can describe these cognitive abilities only with regard to their function under very specific, well-defined circumstances, but not independently of those circumstances.

An important practical implication of this distinction between "implicit knowledge" and "cognitive ability" is that both can easily be mistaken in empirical research. We may be inclined to say a child *knows* the numbers from 1 to 26 when we see it counting wood blocks, when in fact this ability depends on the presence of the blocks. But also counting without objects can be problematic. Maybe the child can count the numbers in the scientist's lab up to 6, but in its classroom up to 23, or *vice versa* (cf. Roth, 2001). There is no crucial experiment to decide what we know, because we can never know to which degree observable behavior is determined by what is present in the respective environment. Since everything that happens happens necessarily in an environment, it is hard to determine whether and to what degree this environment is relevant for a certain performance. That means, assuming "knowledge" can never be more than a hypothesis; we can only be sure in artificial systems where we define ourselves the instructions according to which those systems work.

The terminological distinction between implicit knowledge and cognitive abilities has some implications for the description of cognitive systems in concrete cases. On the one hand, it is important that for every cognitive ability that we list as relevant for a concrete activity there must be listed also the *object* to which this ability refers since, by definition, no cognitive ability exists independently of something else. On the other hand, it is important to note that it is neither possible nor necessary to list *all* forms of implicit knowledge that a person might have at her or his disposal in a concrete situation. This, however, means that we should introduce a further terminological distinction to describe that sub-set of implicit knowledge forms that is relevant and necessary in a concrete situation.

For this purpose, I would use the term "collateral knowledge" that was introduced by Charles S. Peirce (CP 8.183, 6.338, 8.314; cf. Hoffmann & Roth, 2005). "Collateral" means literally "running side by side" with something else. In our context, collateral knowledge can be defined as those forms of knowledge that remain hidden though being an essential *condition* for a cognitive activity. Peirce used the following example to illustrate the concept (cf. CP 8.178; EP II 493). Suppose somebody says "Napoleon was a lethargic creature." Readers probably will be surprised to hear that the great conqueror should be "lethargic." But the point is, to be surprised by this sentence one must already know who Napoleon was and what he accomplished. If we do not know about Napoleon, we can only guess whether it is the name of a lethargic person. We are surprised only when our collateral knowledge about Napoleon contradicts what is said in this sentence. In our case of the counting child it is evident that we have to presuppose a lot of collateral knowledge to explain its ability to count. It needs collateral knowledge to understand the signs provided by the interviewer—"Count as far as you can," or "Count the wood blocks on the table"—, it needs collateral knowledge to say the numbers in the right order, and so on.

As cognitive abilities are dependent on certain objects, so is collateral knowledge. The implicit knowledge who Napoleon was is *collaterally* given only in concrete situations like hearing the sentence "Napoleon was a lethargic creature." This means that we have to specify also for each form of collateral knowledge which is relevant in a concrete context an "object" to which it is related. The "object" in this case is the process of understanding the sentence since it is with regard to this process that the implicit knowledge of who Napoleon was is collateral.

With regard to the objects, however, that we have to specify for each cognitive ability as well as for each form of collateral knowledge we must reflect the following epistemological problem. Specifying those objects presupposes a sort of "God's eye point of view." We are assuming that we *know* what the objects are that are involved. However, the only thing we can know is that all knowledge about objects depends on the cognitive means that the involved persons have at their disposal, respectively. This kind of skepticism with regard to any ontology is basic for epistemological considerations since Immanuel Kant (CPR). "What there is" is determined, first of all, by our theories and conceptual frameworks (cf. Quine, 1971 <1948>). For example, in order to understand the meaning of the wood blocks involved in the child's cognitive system, it does not matter how we as observers understand those; the only thing that matters is how the child perceives them. Accordingly, Agre & Horswill (1997) use the term "lifeworld" to refer to "an environment described in terms of the customary ways of structuring the activities that take place within it" (p. 114). Those "customary ways" are always related to the acting individuals, respectively. "A lifeworld, then, is not just a physical environment, but the patterned ways in which a physical environment is functionally meaningful within some activity" (ibid.). Based on this, I will talk in the following only about "lifeworld objects" in order to indicate that the *meaning* of those objects is dependent on the respective perspective of the involved persons. This is in accordance with the "lifeworld" concept ("Lebenswelt") as it has been developed with Husserl, Merleau-Ponty, Alfred Schütz, and Habermas (cf. Hoffmann, 2005b). And again: specifying those lifeworld objects can only be done based on hypotheses.

"Object" is supposed to mean here something that is either internally or externally present in a certain situation. Thus, not only external *things* like the wood blocks, or *signs* and *persons*, are objects, but it is also possible that a certain form of knowledge or a certain cognitive ability is the "object" of another one. For example, when we interpret a sign like the sentence about Napoleon we need already to know the sign's meaning, or when we calculate an addition in the head, we need to know the rules of addition, and so on. That means that the distinctions I am proposing here can—and should—be applied recursively. The child's ability to count 26 wood blocks in this concrete situation presupposes, for example, the cognitive ability to understand what the interviewer wants it to do, and the collateral knowledge of what words like "count!" mean. Additionally, it presupposes what Michael Tomasello and his coauthors recently called "shared intentionality," a concept that "refers to collaborative interactions in which participants have a shared goal (shared commitment) and coordinated action roles for pursuing that shared goal" (Tomasello et al., 2005, p. 680). Both these conditions of shared intentionality are essential for any form of communication and collaboration, and they are obviously fulfilled in the interview situation described by Hasemann. Otherwise, there would not be any result within a certain range of expectations. Shared intentionality is essential for what I would call *social cognitive systems*. Besides those, there are other, non-social cognitive systems; for example those that are involved when we read a book, or try to solve a problem for ourselves.

Whatever the objects within cognitive systems are, they are present only as *lifeworld objects* in the sense defined above. We know since Peirce that the meaning of a sign depends on its interpretation, and it is important to note that in a cognitive system like the one discussed here the interviewer and the child are not simply "persons." Each of them will be perceived by the other as representing certain characteristics. The interviewer represents for a small child an *authority*, the child is from the interviewer's point of view someone she is responsible for in this situation, and so on. Since this sort of perspectival perception of *objects*—that is of persons, things, signs, and knowledge forms—is obviously important for the cognition that takes place in situations like the counting of wood blocks, I will baptize this phenomenon the *lifeworld dependency of cognition*.

To summarize all these terminological distinctions we can say that every cognitive system can be reconstructed according to the following general scheme that combines two fundamental distinctions: that between *cognitive relations* and *lifeworld objects* on the one hand, and that between *cognitive abilities* and *collateral knowledge* on the other (cf. Table 1). Every cognitive system must be reconstructed from a 1. person's point of view ("I," "we") since, on the one hand, the distinction between what is given as cognitive ability and what as collateral knowledge may vary from person to person and, on the other hand, each lifeworld object is dependent on a specific interpretation as performed by a specific person. How to fill in each of the four fields in the table depends of course on the observer's point of view and specific interests.

The child's	***cognitive relations (R)***	***lifeworld objects (O)***
cognitive abilities (A)	AR.1. to understand her or his own role with regard to AO.1. AR.2. to accept, being motivated by, AO.2. AR.3. to perform the task with regard to AO.3. AR.4. ...	AO.1. the interviewer AO.2. the common goal, shared commitment (Tomasello et al., 2005) AO.3. the wood blocks AO.4. ...
collateral knowledge (K)	KR.1. to understand KO.1. KR.2. ...	KO.1. the signs that are needed to constitute and maintain shared intentionality: a sentence describing what to do ("Count them!"), and gazes, mimics, gestures etc. to focus attention and to signal understanding, questions, etc. KO.2. ...

Table 1: The child's cognitive system when counting 26 wood blocks

WORKING WITH THINGS AND SIGNS

Let me clarify the terminological distinctions I am proposing here with a further example that will lead us, at the same time, to the role that signs and representations play in learning processes. Klaus Hasemann reports a study with children in their last Kindergarten year, half a year before schooling with systematic mathematics education began. The kids were shown the picture of a birthday cake (Figure 1, the original was bigger and colored), and the following story was told:

> Yesterday was Kathrin's birthday. She became nine years old. For her party, her Mom had baked a cake. She put nine candles on the cake [*interviewer points at the cake with the nine candles*]. During the party, the candles were lighted, and at the end Kathrin was allowed to blow them out. After blowing once, still [*interviewer emphasizes*] five candles burned. How many candles did Kathrin blow out? (Hasemann, 2003, p. 36; my translation from the German).

As Hasemann reports, some children were able to solve the problem, others were not. More interesting, however, was the kind of strategies the children used. Since the candles were ordered in a circle, and since apparently all children used a sort of counting strategy, it was nearly impossible to solve the task without any auxiliary means. Some of the kids laid a pencil on the picture to mark the candle where they began to count, some counted backwards, others in two steps forward, etc. Especially interesting were those cases where the children used their fingers as representational means to represent the problem externally.

Figure 1: How many birthday candles are still burning when 5 are blown out? (from Hasemann 2003, p.36)

Interviewer:	... and how many candles did she blow out?
Annika:	*[stares long at the cake]* 4.
Interviewer:	How did you calculate that?
Annika:	I did it with my hands.
Interviewer:	Counted under the table! But then we do not see anything. It's OK to show it. Let us see how you did it.
Annika:	At first, I made the nine [*she shows 9 fingers*]. Then I ... then I removed the five. And then, afterwards, I counted. (Hasemann, 2003, p. 38; my translation)

Hasemann points out that this strategy is remarkable in two respects: First, Annika uses a representation of the objects to count which was neither provided by the picture nor by the situation; that means, she *created* this representation by herself (or applied the habit of doing so creatively from other contexts). Second, she is performing here the first step to a *general* solution, that is to a strategy that can be applied in *any* problem solving case of this sort. Without referring *directly* to concrete objects, or pictures of those objects, she creates a *representation* of this problem situation that can be manipulated independently of—but representatively for—the problem in question. For Hasemann, further steps on this way were "building pure mental situation models, the representation of the situation as relation between numbers, and—finally—its formalization in a calculation like 9 – 5 = x or 5 + x = 9" (ibid.; my translation).

Obviously, the last strategy represents the way we as adults would do it. Being able to answer "4" after a second means that we have a set of capacities at our disposal which comprises first of all *collateral knowledge* according to my definition above; cognitive abilities are involved when there is a direct connection to elements of the environment. The cognitive system as a whole can be reconstructed—hypothetically—as in Table 2.

An adult's	***cognitive relations (R)***	***lifeworld objects (O)***
cognitive abilities (A)	AR.1. to understand her or his own role with regard to AO.1. AR.2. to accept, being motivated by, AO.2. AR.3. ...	AO.1. the interviewer AO.2. the common goal, shared commitment AO.3. ...
collateral knowledge (K)	KR.1. to understand KO.1. KR.2. to understand, based on KR.1., KO.2. KR.3. knowledge of KO.3. KR.4. to identify KO.3. KR.5. knowing how to do KO.5. KR.6. knowing how to do KO.6. KR.7. to understand KO.7. KR.8. ...	KO.1. the picture of the cake, signs the interviewer uses to formulate the task and to constitute and maintain shared intentionality KO.2. the task KO.3. basic algebra as the representational system in which tasks like 9 – 5 can be represented KO.4. KR.3. as adequate for KO.2. KO.5. to translate the real-life task into an algebraic task, or more generally: to represent a problem that is given in one representational system (natural language, pictures, etc.) by means of another one KO.6. to solve 9 – 5 KO.7. the fact that the result of 9 – 5 is at the same time the result of KO.2., that is: the fact that the result obtained in the chosen representational system can be "re-represented" to the original one KO.8. ...

Table 2: An adult's cognitive system when calculating 9–5 without external means

If we observe somebody answering "4" after a second we can assume that he or she possesses the capacities listed here, although this person would hardly be able to confirm this assumption since most parts of the process will be unconsciously performed. Nevertheless, every element listed seems to be necessary to solve the problem.

If we assume that the correct answer is indeed based on what is listed in Table 2, we can say that the two cognitive abilities and the eight forms of collateral knowledge together with their respective lifeworld objects constitute again a "cognitive system."

An adult would need all of them to produce the right answer in this situation. Whatever we are doing is done by means, and within, a "cognitive system." To reconstruct such a system we must first identify the cognitive relations and objects that are relevant for a certain person—and *how* they are relevant for this person—and, second, we must decide for each cognitive relation to those objects whether it is present in the form of *collateral knowledge* (a general ability that is not influenced by environmental factors) or in the form of a *cognitive ability* (dependent on something else).

If we apply this distinction between collateral knowledge and cognitive ability to the research results reported by Hasemann and Reber, it should be obvious that developing a cognitive ability regarding a certain type of tasks is a *precondition* for developing implicit knowledge for those cases. If this assumption is true, it would have important consequences not only for theories of learning, but also for the practice of teaching. It would mean, namely, that *working with things*—concrete objects or representations—is far more important for the development of knowledge than anything else.

This assumption fits to a variety of different approaches that have been developed in psychology and cognitive and educational sciences, but also in philosophy. Consider for example the discussions on "cognitive apprenticeship" and "scaffolding" (cf. Collins et al., 1989; Brown et al., 1989; Rogoff, 1989; Lave, 1993; Lave, 1997); or those on "enactivism" which emphasise in the tradition of Jerome Bruner the relevance of bodily activities for learning processes (Bruner, 1960, 1966; Lakoff & Núñez, 2000; Ernest, 2006). Already John Dewey's "pragmatic" theory "of the method of knowing" is based on the idea that there is a "continuity of knowing with an activity which purposely modifies the environment."[1] And more than a century earlier Hegel developed in his dialectic of "lordship and bondage" the idea that it is "through work" that "the bondsman becomes conscious of what he truly is" (Hegel, 1977 <1807>, p. 118). Working on the things his lord possesses mirrors not only his "alienated existence," but is at the same time the very precondition to develop self-consciousness (ibid., p. 119).

Within the framework developed so far, two main research questions become visible that should be of crucial importance for educational sciences. The first question is how exactly activities can promote the development of cognitive abilities. How is it possible that the child develops the cognitive ability to count 26 wood blocks without being able to count the numbers themselves so far? The second question is how we can bridge the gap between a certain cognitive ability that is dependent on a certain situation on the one hand, and a corresponding implicit knowledge that is *in*dependent of those situations on the other. How is it possible to move from Annika's cognitive ability to solve the candle problem by means of her fingers to a cognitive system that is mainly based on collateral knowledge as described in Table 2?

FROM THE FINGERS AS REPRESENTATIONAL SYSTEM TO BASIC ALGEBRA

Let me start with the second question. For that, it might be useful to analyze, in a first step, the cognitive system that becomes visible in Annika's strategy to solve the candle task by means of her fingers (cf. Table 3). In order to explain the development from her cognitive abilities to an adult's knowledge, we can then compare the Tables 2 and 3.

Annika's	***cognitive relations (R)***	***lifeworld objects (O)***
cognitive abilities (A)	AR.1. to understand her own role with regard to AO.1.	AO.1. the interviewer
	AR.2. to accept, being motivated by, AO.2.	AO.2. the common goal, shared commitment
	AR.3. to identify AO.3.	AO.3. KR.3. as adequate for KO.2.
	AR.4. being able to do AO.4.	AO.4. to represent the problem situation by means of her fingers (in a Peircean theory of learning and creativity, this would be the ability to perform the first step of what Peirce called "diagrammatic reasoning": the *construction* of a "diagram," that is the construction of a representation which mirrors the elements of an object world and their relational structure by means of a representational system; cf. Hoffmann, 2005)
	AR.5. being able to do AO.5.	AO.5. to manipulate the fingers in a way that mirrors the process described in the problem situation, that is the blowing out of five candles (this corresponds to the second step of Peirce's diagrammatic reasoning: *experimenting* with the diagram)
	AR.6. being able to do AO.6.	AO.6. to *observe* the results of her finger experiment, that is the ability to see that four fingers remain when five are removed (this would be the third step of diagrammatic reasoning)
	AR.7. being able to do AO.7.	AO.7. to understand that the result of the finger calculation is at the same time the result of the real-life problem (KO.2.)
	AR.8. ...	AO.8. ...
collateral knowledge (K)	KR.1. to understand KO.1.	KO.1. the picture of the cake, signs the interviewer uses to formulate the task and to constitute and maintain shared intentionality
	KR.2. to understand, based on KR.1., KO.2.	KO.2. the task
	KR.3. knowledge of KO.3.	KO.3. her fingers
	KR.4. ...	KO.4. ...

Table 3: Annika's cognitive system when calculating 9–5 by means of her fingers

My suggestion to reconstruct Annika's cognitive system as mainly based on cognitive abilities rests on a certain caution. We do not know exactly to which degree her ability to calculate on the fingers is "independent of her environment." But this is an interesting question. What becomes visible here is that there is no clear-cut distinction between a "cognitive ability" and a corresponding form of "collateral knowledge"; rather, there is a continuity between both, a continuity that seems to be an essential precondition for the possibility of learning. If I knew that Annika is a trained finger calculator, I would not hesitate to call her ability a form of collateral knowledge. Although her fingers can be considered part of her environment, a trained finger calculator would not be influenced by factors like cold fingers, or fingers wearing gloves, and so on. In this case, finger calculation would be as abstract as the calculation in the head. It would be abstract in the sense of being clearly separated from the task in question, and of providing a general *possibility*. Being able to use a representational system means being able to abstract a problem from its context.

Comparing Tables 2 and 3, we can see not only that Annika's cognitive abilities have been established as knowledge forms, but also that the essential difference between an adult's cognitive activities and those of Annika is that both are using a different representational system into which they translate the problem: either basic algebra or the ten fingers. Besides that, there is only one minor difference which depends on this concrete example. In Annika's lifeworld-based cognitive system we have to distinguish between her *experimenting* with the fingers (AR.5.), that is the process of taking away five fingers from the original nine, and her *observation* of the result (AR.6.), that is to see that four fingers are remaining. Both these steps are combined with an adult in KR.6. This, however, is only the case when the task can be resolved "in the head" as with this simple task; every more complicated task for which we use a representation on a piece of paper, or a calculator, presupposes again the two different activities that Annika performs: experimentation and observation.

However, the *similarities* between Annika's cognitive system and the more intramental cognitive system an adult would activate when facing the candle task are more astonishing than all the differences. These similarities offer a hint at how to answer the second research question formulated above: the question how to bridge the gap between a situation dependent cognitive ability and a corresponding abstract, collateral knowledge. Based on our hypothesis that each form of knowledge presupposes a corresponding cognitive ability that is functionally connected with concrete activities, we can say that the development from Annika's cognitive system (Table 3) to a corresponding cognitive system of an adult (Table 2) is essentially a process of "internalization." The calculation on the fingers as a concrete and visible activity gets replaced by the abstract, algebraic calculation. As Edwin Hutchins observed in *Cognition in the wild*, those processes of internalization are mainly based on repetition:

> With experience we learn about the regularities of the world of external symbolic tokens and we form mental models of the behaviors of these symbolic tokens that permit us to perform the manipulations and to anticipate the possible manipulations. With even more experience, we can imagine the symbolic world and apply our knowledge, gained from interactions with real physical symbol tokens, to the manipulation of the imagined symbolic worlds. (Hutchins, 1995, pp. 292-293)

Internalization based on repeated experience can explain how we can come from a situation dependent cognitive ability to a corresponding abstract and implicit knowledge. This explanation corresponds to what Lev Vygotsky formulated already about 70 years ago with regard to the priority of *socially* relevant cognitive abilities in his "general genetic law of cultural development":

> We can formulate the general genetic law of cultural development as follows: every function in the cultural development of the child appears twice, in two planes, first, the social, then the psychological, first between people as an intermental category, then within the child as an intramental category. This pertains equally to voluntary attention, to logical memory, to the formation of concepts, and to the development of the will. We are justified in considering the thesis presented as a law, but it is understood that the transition from outside inward transforms the process itself, changes its structure and functions. Genetically, social relations, real relations to people, stand behind all the higher functions and their relations. From this, one of the basic principles of our will is the principle of division of functions among people, the division into two of what is now merged into one, the experimental unfolding of a higher mental process into the drama that occurs among people. (Vygotskij, 1997, p. 106)

This quote is taken from a passage in Vygotsky's *History of the Development of Higher Mental Functions* where he describes how an infant learns the pointing gesture in interaction with its mother. Vygotsky emphasizes here that the development of the pointing gesture "plays an exceptionally important role in the development of speech in the child and is, to a significant degree, the ancient basis for all higher forms of behavior" (p. 104). What makes this gesture so exceptional is that the ability to use it, and to understand it when used by others, can indeed be seen as the fundament for using and understanding *signs* in general. The ability to point not only seems to be what distinguishes us from our nearest primate relatives, the great apes (Tomasello, 2006), but it can also be seen as the paradigm of *semiotic competence* in general. Who is able to interpret the pointing finger as a *sign*—that is, who knows that the point in pointing is not the pointing finger but the *object* pointed at—knows already two basic ideas of semiotics: first, that a "sign" is "something which stands to somebody for something" (Peirce, CP 2.228) and, second, that any ordinary physical object can *function* as a sign if interpreted as such.

However, Vygotsky's distinction between *two* appearances of cognitive functions —on the social and on the intramental level—may not be the end of the story. In his own semiotic approach Vygotsky focused on the idea that the "emergence of uniquely human, higher mental functions such as thinking, voluntary attention, and logical memory" can be explained by the fact that the signs somebody uses "in social interaction to control others' activity become a means for controlling one's own activity."[2] This *self-regulatory function of signs* is exactly what is essential when Annika uses her fingers to calculate 9 – 5. Watching her fingers allows her to *regulate* and *monitor* her own "mental" processes. In her case, however, this cognitive activity of working with signs and representations is already separated from social interaction. She could do the same also independently of the social situation of the interview. Based on that, I would suggest that "every function in the cultural development of the child appears" not only twice, but three times: first, on the social level when somebody shows the child how to use the fingers to represent arithmetical problems; second, when the child for herself uses this representational system as an external tool to regulate her own cognitive processes; and third, when she is able to solve the task "intramentally" as described in Table 2.

THE CONSTRAINING POWER OF EXTERNALS: PEOPLE, SIGNS, AND THINGS

Let me turn now to the first research question mentioned above which referred to the observations formulated by Hasemann and Reber that—formulated in my terminology—*cognitive abilities* like being able to count 26 wood blocks are always ahead of *implicit knowledge* as it becomes visible when a child can count 6 numbers. Based on Vygotsky's idea of a self-regulatory function of signs we can assume that not only signs can fulfill this function, but things as well. Rather than assuming that in counting 26 wood blocks a ready-made *knowledge* of the ordered sequence of the numbers from 1 to 26 is at the child's disposal, we can assume that it is the ordered sequence of the *wood blocks* that constrains the way the child is operating with the numbers. The child in Hasemann's experiment seems to be in a state where it knows the names of the numbers, and has also a more or less vague idea of their sequence, but needs something *to observe* in order to organize the activity of counting.

Hutchins has shown with many examples how important the constraining and regulatory power of the external world is. Most important are other *people* who direct our attention, define the horizon of observation, teach us activities and terminologies, monitor what we are doing, hint at errors, and give examples of how to do things (pp. 263–285). But then there are also the *tools* that we do not only use, but that affect at the same time our cognitive activities.

> The design of a tool may change the horizon of observation of those in the vicinity of the tool. For example, because the navigation chart is an explicit graphical depiction of position and motion

> [on a ship, M.H.], it is easy to "see" certain aspects of solutions. The chart representation presents the relevant information in a form such that much of the work can be done on the basis of perceptual inferences. (Hutchins, 1995, p. 270)

Within this framework, representations are first of all cognitive tools, that is external parts of cognitive systems that—at the same time—constrain and promote cognitive activities so that they develop in a certain direction. If there is a work sheet with labeled blank spaces to fill out, then the structure of the form regulates to a certain extent the structure of activities we should perform to cope with a task (pp. 280, 294).

> The chief's use of the form is both a way to organize his own behavior and an example to the novice of a way to use such a resource to organize behavior. Given the form, the novice might now be able to reproduce the chief's use of the form to organize his own behavior without the chief's being present. The performance of the task also provides the novice with the experience of the task and the sequence of actions that can accomplish it. We might imagine that, with additional experience, the novice would be able to remember the words of the chief's queries, remember the meanings of the words, and remember physical actions that went into the satisfaction of those queries. (Hutchins, 1995, p. 281)

Hutchins highlights especially the relevance of social relations, that is relations in which "each provides the others, and the others provide each, with constraints on the organization of their activities" (p. 282). But also tools that we are using independently of social relations have this constraining and promoting power, as well as ordinary things that may be present in our environment. Consider for example how the environment guides you in finding a way you are not really familiar with but that you traveled before. Although it might be impossible for you to formulate beforehand an instruction where to go—that is, your *knowledge* is limited—some features of the environment that become visible as you go remind you exactly where to go next at each position. The "knowledge" where to go is more located in the world outside than in your mind (cf. Hoffmann & Roth, 2005, p. 124). This is exactly what the "lifeworld dependency of cognition" is supposed to mean. Accordingly, in what has been called "cognitive autopoiesis," agents deliberately "structure their environments in order to provide the conditions for their own cognitive activities" (Agre & Horswill, 1997, p. 138).

In philosophy, the function of external things for the development of knowledge was discussed extensively by Charles S. Peirce already a century ago. Peirce, the founder of a tradition in semiotics that focuses primarily on epistemological and pragmatic aspects of signs, is interested in the question how we can learn by means of external representations, that is how signs and representations regulate our cognitive activities and the development of cognitive abilities. I mentioned in Table 3 above the main concept he introduced with regard to this: "diagrammatic reasoning."[3] Peirce developed this concept to describe the specific nature of "The Reasoning of

Mathematics." In the intellectual autobiography he wrote for his so-called "Carnegie Application" in 1902, he summarizes his views as follows:

The first things I found out were that all mathematical reasoning is diagrammatic and that all necessary reasoning is mathematical reasoning, no matter how simple it may be. By diagrammatic reasoning, I mean reasoning which *constructs a diagram* according to a precept expressed in general terms, *performs experiments* upon this diagram, *notes their results*, *assures itself* that similar experiments performed upon any diagram constructed according to the same precept would have the same results, and *expresses* this in general terms. This was a discovery of no little importance, showing, as it does, that all knowledge without exception comes from observation. (Peirce, NEM IV, pp. 47-48; my italics)

The regulative and cognition-promoting function of diagrammatic reasoning becomes visible when Peirce discusses its relation to his "pragmatism," another influential approach he is famous for having founded. For Peirce, "pragmatism"—or "pragmaticism" as he called it later to distance himself from followers like William James and Ferdinand Schiller (Peirce, EP II, p. 335)—is first of all a *method* to clarify the meaning of concepts or "symbols," that is of those signs whose meaning depends on conventions or habits of usage. His famous "pragmatic maxim" demands, as he wrote in a manuscript in 1905:

In order to ascertain the meaning of an intellectual conception one should consider what practical consequences might conceivably result by necessity from the truth of that conception; and the sum of these consequences will constitute the entire meaning of the conception. (Peirce, CP 5.9; Peirce's italics. Cf. also CP 5.438)

Performing such a "practical consideration"—which includes the practice of thinking about "general ideas" (cf. CP 5.3)—means first of all to consider possible applications of a concept in a way that an idea of such an application can force you to modify an initial assumption. Thinking about the meaning of a concept in form of possible *experiences* constrains and regulates the development of our mental representation of this concept. Exactly this is what Peirce sees most clearly realized in the "reasoning of mathematics" (CP 5.8):

Such reasonings and all reasonings turn upon the idea that if one exerts certain kinds of volition, one will undergo in return certain compulsory perceptions. Now this sort of consideration, namely, that certain lines of conduct will entail certain kinds of inevitable experiences is what is called a "practical consideration." (Peirce, CP 5.9)

This is Peirce's version of Vygotsky's idea that the signs we use become a means for controlling and regulating our own cognitive activities. His explanation of this possibility, however, is different. What is essential for diagrammatic reasoning as performed in mathematics is what he calls in the quote above "inevitable experiences" that lead to "certain compulsory perceptions." The reason for that is that any *construction* of a diagram—defined as "a representamen which is predominantly an icon of relations," what implies that also a sentence, or a text, can be a "diagram"—and any *experiment* with such a diagram is constrained and regulated by the rules and conventions of the "system of representation" we choose to represent something (CP 4.418). You can prove the theorem that the triangle's inner angles sum up to 180° if you perform the proof by means of Euclidean geometry as your representational system, but you cannot do that if you choose a non-Euclidean geometry (cf. Hoffmann, 2004). The former provides specific means like the parallel axiom that can be used to perform proofs which are not available in the latter ones. Every system of representation is defined by "certain general permissions to modify the image [or diagram, M.H.], as well as certain general assumptions that certain things are impossible" (Peirce, CP 5.8). These rules and conventions determine the outcome of diagrammatic reasoning. "Under the permissions, certain experiments are performed upon the image, and the assumed impossibilities involve their always resulting in the same general way" (ibid.). Since the outcome of experimenting with diagrams is determined by the representational system used, diagrammatic reasoning necessarily leads to "inevitable experiences" that regulate the mathematician's cognitive activities "from outside." Additionally, if those experiments lead to more problems than they can solve, the process of diagrammatic reasoning will challenge us either to look for a better representation or to modify our representational system (cf. Hoffmann, forthcoming).

This way, diagrammatic reasoning can be a powerful tool for both to regulate our cognitive activities and to stimulate learning and creativity. For Annika her fingers already provide a *general* system of representation—she could calculate all sorts of arithmetic tasks on them.[4] But there will be a time when she realizes how much more powerful algebra can be. On her way, working with signs and representations will be of extraordinary importance. Thus, Willi Dörfler (forthcoming) argues with regard to mathematics education that diagrammatic reasoning with its focus on signs, and activities performed on signs, should replace, or at least complement, the traditional orientation at "abstract objects." Participating in the *practice of diagrammatic reasoning* is more important for learning and creativity than the rote learning of concepts and strategies.

And this is true not only for mathematics education. While Peirce developed the concept of diagrammatic reasoning especially with regard to mathematics and necessary reasoning, there are attempts to extend its applicability in a way that it can be used for a general theory of scientific discovery and learning (Hoffmann, forthcoming). The general message is: Although it is possible to solve problems without representational means, it should be absolutely clear that the ability to work with signs and representations is one of the most powerful capacities humans have developed. The earlier children learn how to solve problems by means of representations, the better.

COGNITIVE SYSTEMS AS SEMIOTIC SYSTEMS

My main argument in this paper is that in order to explain the possibility of learning we have to analyze not only what happens in an individual's mind, but *cognitive systems* that include other people, things, and signs. Working with concrete objects, and with representations of those objects and relations among them, both regulates, constrains, and stimulates our cognitive abilities and allows us then—by means of repetition and internalization—to transform those cognitive abilities into knowledge.

While it seems to be convincing that we learn indeed primarily from interactions with other people and from activities with things and signs, a remaining problem is that, by now, we do not have a clear understanding of how these three resources are connected in learning processes. Falk Seeger (forthcoming; cf. Seeger, 2005) recently hinted at the fact that the two semiotic approaches that can be labeled as Peircean *versus* Vygotskyian are still mainly discussed side by side, but not within an encompassing, theoretical framework. Referring to both these traditions, he talks about the "necessary unity of semiotic processes relating to objects and semiotic processes relating to persons on a social plane." Indeed, while Peirce is primarily interested in how we can learn something about the world by means of diagrammatic reasoning, Vygotsky's starting point is the shift from regulating social relations by means of signs to regulating one's own thinking processes. Seeger argues that we can not get one of these possibilities without the other. What we need, therefore, is a theory that elaborates the *simultaneity* of those conditions of learning that refer to things on the one hand, and those referring to social relations and the role of individuals and interactions between those on the other (ibid.).

Seeger's argument is convincing, even if both the examples we discussed above—the counting child and Annika's finger calculation—only seem to refer either to things or to representations, not to social relations. This impression, however, is misleading. Since all the cognitive systems I reconstructed above (Tables 1 to 3) are embedded in the *social* situation of an interview, they are based on what Tomasello and his coauthors called "shared intentionality" (Tomasello et al., 2005). Although Annika is performing her finger calculation as private as possible—under the table!—her eagerness to do her best to answer the interviewer's question can only be explained by her eagerness to maintain the *social relation* in which she finds herself in the setting of the interview. It is a well-known fact that very small infants are already "extremely sensitive to social contingencies" (Tomasello et al., 2005, p. 681). They engage in what has been called "protoconversations," that is in "social interactions in which the adult and infant look, touch, smile, and vocalize toward each other in turn-taking sequences"; and their early social interactions "clearly show mutual responsiveness on the behavioral level" to maintain those protoconversations (ibid.).

Tomasello and his coauthors claim that shared intentionality is so basic that it is even the fundament of language acquisition. This claim has been hotly debated in the issue of *Behavioral and Brain Sciences* in which their paper was published. However, it seems without any doubt that mother and child must use *some* signs—gaze, mimics, body movement, and so on—to *establish* shared intentionality first of all. Developing the ability to engage in shared intentionality on the one hand and at least most rudimentary forms of semiotic competence on the other can hardly be separated into two independent processes. They are mutually dependent.

Based on this consideration, Falk Seeger's suggestion to organize the relationship between Peirce's object-oriented approach and Vygotsky's orientation at social interaction around the sign concept seems to be well-taken (Seeger, forthcoming). Putting the sign in the middle seems natural since in both these approaches the sign has a mediating function: it mediates between an "interpretant" and an "object" with Peirce, and between individuals with Vygotsky. While Vygotsky's semiotics focuses on the idea that we are using signs as a *means* to regulate, first, social relations and, second, our own thinking processes, Peirce defines his concept of a sign as follows:

> A sign, or *representamen*, is something which stands to somebody for something in some respect or capacity. It addresses somebody, that is, creates in the mind of that person an equivalent sign, or perhaps a more developed sign. That sign which it creates I call the *interpretant* of the first sign. The sign stands for something, its *object*. (Peirce CP 2.228; cf. Figure 2)

As one of the most impressive examples of how the Peircean and the Vygotskyian sign functions can be combined in learning processes, Seeger hints at Vygotsky's description of how an infant learns the pointing gesture (Vygotskij, 1997, pp. 104–106). As already mentioned, for Vygotsky this process "is, to a significant degree, the ancient basis for all higher forms of behavior" (p. 104). He explains its development as based on a child's *intention* to grasp an object that is too far way and the mother's *interpretation* of the child's finger movements *as pointing*. The essential step, however, is the child's insight—at a certain point—that it can *use* its finger movement to manipulate its mother's behavior.

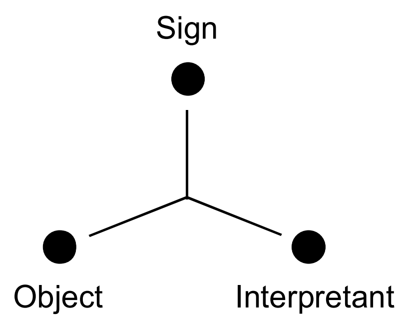

Figure 2: Peirce's triadic sign relation

> In this way, the child is the last one to recognize his gesture. Its significance and function are initially made up of an objective situation and then by the people around the child. The pointing gesture most likely begins to indicate by movement what is understood by others and only later becomes a direction for the child himself. (Vygotskij, 1997, p. 105)

If we focus only on the *elements* which are part of a cognitive system after the development of the pointing gesture is completed, we can reconstruct this system in its most basic form as in Figure 3. By contrast to Peirce's *triadic* sign relation (cf. Figure 2), I call this a *tetradic* relation since four constitutive elements are involved. This model can be read from three different points of view: from the sign user's, the addressee's, and an observer's perspective.

1. A *sign user* uses a sign in order both to direct the attention of an addressee to a certain object and to signal at the same time that she herself is focusing on this object.
2. An *addressee* perceives a sign as used by a sign user, first, as addressed to her, second, as a means to direct her attention to a certain object and, third, as signaling the sign user's intention regarding this object.

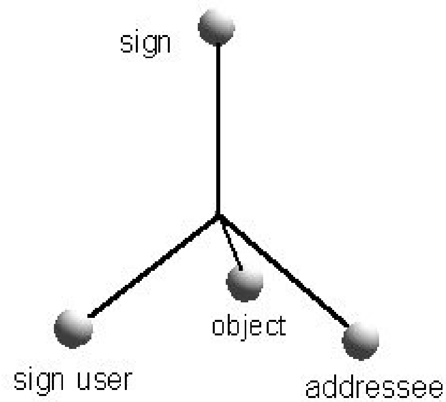

Figure 3: A tetradic, sign-mediated cognitive system

3. An *observer* perceives a sign as a means that is supposed to fulfill both the functions described in (1) and (2). An archeologist, for instance, might find a sign that she interprets from an assumed sign user's point of view.

The sign in this cognitive system mediates simultaneously the relation to an object and the relation between the persons involved. From my point of view, Figure 3 shows a general possibility to model the "unity of semiotic processes relating to objects and semiotic processes relating to persons" for whose necessity Falk Seeger argues. This unity is necessary because all four elements must be present at the same time to provide the possibility of a *social cognitive system*.

Figure 3 can be used not only to model the pointing gesture in which the "object" would be something the finger points at, but for modelling any social cognitive system. In protoconversation a child's smile can be interpreted as a sign that indicates as its object the child's eagerness to create a status of shared intentionality with her mother; in an interviewer's demand "Count these wood blocks" this sentence would be a sign that represents what the she wants the child to do; in Annika's demonstration of how she solved the candle task her fingers are representing the candles on the cake for the interviewer and for herself; and so on. In each of these cases, the general model described in Figure 3 can be specified by distinguishing for both sign user and addressee the cognitive relations and lifeworld objects that I elaborated exemplary in Tables 1 to 3.

Of course, cognitive systems can be more complex than in the examples I discussed here; for instance, there might be more persons involved, or different objects have to be related. However complex the ontology of a cognitive system might be, it is *the function of the sign to bind all this together* in a concrete situation. The sign determines which object, set of objects, relation between objects—be they abstract or concrete—is meant, we choose a sign according to the cognitive capacities of the person the sign is supposed to address (e.g. adult *versus* child), and whenever someone wants to understand our intentions, she or he has to interpret *signs*: the words we are using, diagrams we are constructing, gestures, gazes, mimics, body language, voice modulation, and so on. Signs are the fundament to constitute shared intentionality and communication, they are the essential mediators.

For this reason, I would argue that cognitive systems are first of all semiotic systems: systems mediated and constituted by signs and representations. It should be important, based on this, to use semiotic approaches and theories in cognitive science and in educational sciences as well. What has been done in semiotics especially regarding the conditions of sign interpretation can shed new light on problems we are facing when we try to explain the possibility of learning (e.g. Sáenz-Ludlow, 2006; Otte, 2006, Duval, 2006; Presmeg, 2006; Morgan, 2006; Hoffmann & Roth, forthcoming).

Daniel Goleman argues in his new book *Social Intelligence* that the primary function of cognition is to regulate social interaction: "we are wired to connect" (Goleman, 2006, p. 4). It is important to note, however, that this interaction and connection could never be possible without signs.

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NOTES

1 Dewey, 1976 <1916>, MW 9:353-54. Dewey's pragmatism, again, rests on Peirce's; cf. for the relevance of activity in Peirce Colapietro, 2005 and Anderson, 2005.

2 Wertsch, 1994, p. 1159; cf. Vygotskii, 1978 <1960>, pp. 38-57, 79-91; Seeger, 1998.

3 Cf. Stjernfelt, 2000; Dörfler, 2004, 2005, forthcoming; Hoffmann, 2003, 2004, 2005a.

4 Ifrah, 1991 <1981> shows that over centuries finger calculation was the primary tool in all big civilizations (pp. 79-109).