Working Paper Series

Working Paper #24

Cognitive Conditions of Diagrammatic Reasoning

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February 2007

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SPECIAL ISSUE ON PEIRCEAN DIAGRAMMATICAL LOGIC

edited by Joao Queiroz and Frederik Stjernfelt

in

SEMIOTICA

February 21, 2007

Cognitive Conditions of Diagrammatic Reasoning*

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Abstract

In the first part of this paper, I delineate Peirce's general concept of diagrammatic reasoning from other usages of the term that focus either on diagrammatic systems as developed in logic and AI or on reasoning with mental models. The main function of Peirce's form of diagrammatic reasoning is to facilitate individual or social thinking processes in situations that are too complex to be coped with exclusively by internal cognitive means. I provide a diagrammatic definition of diagrammatic reasoning that emphasizes the construction of, and experimentation with, external representations based on the rules and conventions of a chosen representation system. The second part starts with a summary of empirical research regarding cognitive effects of working with diagrams and a critique of approaches that use 'mental models' to explain those effects. The main focus of this section is, however, to elaborate the idea that diagrammatic reasoning should be conceptualized as a case of 'distributed cognition.' Using the mathematics lesson described by Plato in his Meno, *I analyze those cognitive conditions of diagrammatic reasoning that are relevant in this case.*

Keywords: Diagrammatic reasoning, distributed cognition, mental models, learning, scaffolding, creativity

1. What is 'diagrammatic reasoning'?

The concept of 'diagrammatic reasoning' was first introduced, as far as I can see, by John Venn in his article 'On the Diagrammatic and Mechanical Representations of Propositions and Reasoning' (Venn 1880). Venn's idea was to develop with his 'Venn diagrams'—in the tradition of Euler's 'circles' (Euler 1768)—a graphical *alternative* to sentential and algebraic forms to represent logical relations. Charles Peirce continued this work with his so-called 'Existential Graphs' (Peirce 1909; Roberts 1973; Shin 2002). 'The System of Existential Graphs,' he says, 'greatly facilitates the solution of problems of Logic' (Peirce CP 4.571).

With regard to Peirce, however, it would be a mistake to identify diagrammatic reasoning with reasoning by means of Existential Graphs. For him, diagrammatic reasoning is more generally *any* form of 'valid necessary reasoning' (CP 1.54; 5.162). The logical operations that are possible by means of his Existential Graphs are only *one* form of diagrammatic reasoning besides others. Even algebraic forms can be 'diagrams' (cf. CP 2.778; EP II 13), as well as sentences like 'Ezekiel loveth Huldah' (Peirce EP II 17). According to Peirce's semiotics where all these terms are precisely defined, ‘diagrams’ form a sub-set of ‘icons.’ The basic idea of ‘icons,’ however, is neither that they have a graphical or pictorial form, nor that they are similar to the objects they represent—although both is the case with icons like a photograph, a ‘piece of mimicry,’ or a footprint—but that they represent *relations:*

> Many diagrams resemble their objects not at all in looks; it is only in respect to the relations of their parts that their likeliness consists. ... When, in algebra, we write equations under one another in a regular array, especially when we put resembling letters for corresponding coefficients, the array is an icon. ... In fact, every algebraic equation is an icon, in so far as it *exhibits*, by means of the algebraic signs (which are not themselves icons), the relations of the quantities concerned. (Peirce EP II 13)

The specific difference of ‘diagrams’ in relation to other icons is that they are carried out according to certain ‘precepts’ (CP 2.216; NEM IV 47), that is according to the rules and conventions that are defined in a certain ‘system of representation’:

> A *diagram* is a representamen which is predominantly an icon of relations and is aided to be so by conventions. Indices are also more or less used. It should be carried out upon a perfectly consistent system of representation, founded upon a simple and easily intelligible basic idea. (Peirce CP 4.418)

A sequence of words like ‘Ezekiel Huldah loveth’ might be interpreted as an icon, if we perceive it as representing a certain relation. But if we read ‘Ezekiel loveth Huldah,’ we have a diagram, because this sign represents a relation that is carried out according to the rules of English grammar.

The fact that, according to Peirce, diagrams must be constructed by means of a certain representational system is essential for an adequate understanding of his concept of diagrammatic reasoning. His Existential Graphs are such a ‘perfectly consistent system of representation.’ Its soundness and completeness has been proven (Zeman 1964; Roberts 1973). However, axiomatic systems in mathematics are also consistent systems of representation, and our everyday languages are of course representational systems as well, although not necessarily ‘consistent’ ones. Peirce, indeed, says that he developed the concept of diagrammatic reasoning to describe the specific nature of ‘The Reasoning of Mathematics.’ In his so-called ‘Carnegie Application,’ he writes about the relevance of his discovery:

> The first things I found out were that all mathematical reasoning is diagrammatic and that all necessary reasoning is mathematical reasoning, no matter how simple it may be. By diagrammatic reasoning, I mean reasoning which *constructs a diagram* according to a precept expressed in general terms, *performs experiments* upon this diagram, *notes their results*, *assures itself* that similar experiments performed upon any diagram constructed according to the same precept would have the same results, and *expresses* this in general terms. This was a discovery of no little importance, showing, as it does, that all knowledge without exception comes from observation. (Peirce NEM IV: 47-48; my italics)

This quote not only provides one of the most precise *definitions* of Peirce's general concept of diagrammatic reasoning, it also says something about one of the main *functions* it is supposed to fulfill: diagrammatic reasoning is a tool to generate knowledge (cf. CP 3.559 f.; 4.530 f.; 4.571). This heuristic function of diagrammatic reasoning is different from the function the Existential Graphs are supposed to fulfill, namely to facilitate 'the solution of problems of Logic' (Peirce CP 4.571).

This duality of two different functions of diagrammatic reasoning that we can find in Peirce's writings is mirrored in a variety of quite different interpretations of the term 'diagrammatic reasoning' we are facing nowadays in several research contexts. The term saw, for example, an impressive renaissance in computer science and logic over the last decades. Starting point was the idea that visual information is easier for humans to handle and plays a more important role in communication and learning than sentential representation systems and algebraic notations. Based on that, diagrammatic modeling of software systems, of knowledge representation, and of proof methods are some of the basic goals in this area (cf. Allwein & Barwise 1996; Anderson et al. 2000; Anderson et al. 2002; Barker-Plummer et al. 2006; Blackwell et al. 2004; Glasgow et al. 1995; Hammer 1995; Hegarty et al. 2002; Molina 2001; Shin 1994; cf. also the overview by Shin & Lemon 2003). The focus in this work is mainly on diagrammatic representational systems in themselves—that is in their soundness, completeness, and usage in proofs.

Another research tradition that is at home mainly in psychology and cognitive science uses the term 'diagrammatic reasoning' to describe the process of interpreting diagrammatic representations (Glasgow et al. 1995; Hegarty 2000; Larkin & Simon 1987). This approach can be related to a huge literature on 'model-based reasoning' and 'mental modeling' (Bauer & Johnson-Laird 1993; Johnson-Laird 1983; Johnson-Laird 1996; Magnani & Nersessian 2002; Magnani et al. 1999; Nersessian 2002). In all these areas, the focus is, first of all, on mental processes.

By contrast to both these main lines of reasoning about diagrammatic reasoning, what I am interested in is what I described above as Peirce's general concept of diagrammatic reasoning. Instead of dividing work on diagrammatic systems on the one hand, and on reasoning on the other, the focus here is on those forms of *interplay* between diagrams and reasoning that *promote human creativity and learning* (cf. Craig et al. 2002; Dörfler 2004, 2005; Giere 2002; Hoffmann 2004, forthcoming-b; Magnani 2002; Stjernfelt 2000). This research can be related to studies on the role of graphical representations in 'external cognition' (Scaife & Rogers 1996). My main interest concerns an analysis of those cognitive processes that are involved when human beings perform diagrammatic reasoning to solve problems, to cope with complexity, to learn something new, or to resolve conflicts. Possible applications of diagrammatic reasoning I have in mind are the following:

- Peirce's description of 'the mathematician's business': to help an engineer, or a business company, or a physicist 'to ascertain what the necessary consequences of possible facts would be' in a situation where the facts are so complicated that these people cannot deal with them in their usual way (Peirce CP 3.559 f.)
- when Maxwell draws a figure to derive the mathematical representation of the electro-magnetic field concept (cf. Nersessian 2002)
- when a child tries to solve an arithmetical problem by means of her fingers (cf. Hoffmann forthcoming-a)
- when Socrates—as described in Plato's *Meno*—helps a boy to discover how to double a square by means of experimenting with figures drawn in the sand (Hoffmann 2003)
- when Computer Supported Argument Visualization (CSAV) tools are used as 'sense-making tools to negotiate understanding in the face of multi-stakeholder, ill-structured problems' (Kirschner et al. 2003)
- when negotiators try to resolve a conflict through Logical Argument Mapping (LAM), a tool that is supposed to change stakeholders' mind-sets by visualizing implicit assumptions and stereotypes (Hoffmann 2005b).

A *general* description of possible applications of diagrammatic reasoning is implied in what I consider here as the main *function* of diagrammatic reasoning: *to facilitate individual or social thinking processes in situations that are too complex to be coped with exclusively by internal cognitive means*. Such a 'facilitation' of thinking processes should be possible based on a variety of characteristics of diagrammatic reasoning. Having something in front of your eyes allows one to

1. reflect on something without being constrained by the limits of one's short-term, or working memory
2. analyze a problem more thoroughly and systematically
3. clarify and coordinate confused ideas about a problem
4. clarify implicit assumptions and background knowledge that might be insufficient or inadequate
5. structure a problem space
6. change perspectives
7. identify 'unintended and unexpected' implications, like the general who sticks pins into a map during a campaign, 'so as to mark each anticipated day's change in the situations of the two armies' (Peirce CP 4.530).
8. play with interpretations (cf. Lindsay 2000; Roth 2004)
9. discover contradictions
10. distinguish the essential from the peripheral

Especially in collaborative settings, diagrammatic reasoning is supposed to

11. focus attention by ‘putting something in the middle’ (Selvin 2003)
12. initiate ‘negotiation of meaning’ regarding the elements used in a diagram and to motivate argumentation (Suthers & Hundhausen 2003)
13. help to see and to explore the multi-perspectivity of a problem (Kanselaar et al. 2003)
14. establish and maintain common ground (Baker et al. 1999)
15. function as an external group memory that stabilizes the continuity of interactions and pushes things forward.

In short, the main idea to facilitate thinking processes by diagrammatic reasoning is that it should be possible to reduce the cognitive load in individual and collaborative problem solving, decision making, and conflict management by means of external representations (cf. van Bruggen et al. 2002; Hoffmann 2005b). Three points are important for this understanding of diagrammatic reasoning. First, by contrast to what could be called ‘diagrammatic communication,’ diagrammatic reasoning is not concerned with representing something we already know. When I draw a map to explain a friend how to drive to a certain location, I would *communicate* by means of a diagram but I would not *reason* with it. Diagrammatic *reasoning* is about problem solving, decision making, knowledge development, and belief change by means of diagrams. However, I do not presuppose a clear cut distinction between diagrammatic communication and diagrammatic reasoning. There might be a continuity between both these possibilities. Reasoning with diagrams might have a communicative function as well—for example in collaborative learning, or deliberative decision making—and it might turn out that what we intend simply to communicate stimulates some interesting reasoning.

The second point is referring to *external* representations. While I am following Peirce regarding his definition of diagrammatic reasoning, I restrict the use of the term to processes in which *external* representations are an irreducible part. Peirce, by contrast, sometimes assumes that diagrammatic reasoning can also be performed ‘in the imagination’ (Peirce NEM I 122; CP 1.443; 4.530). I do not doubt, of course, that it is possible to *imagine* how to determine the side of a square double the size of another square, or to calculate 9–5 without using the fingers, or to estimate the energy I need to move a weight on the longer arm of a balance by means of a ‘mental image’ of this balance; but my point is simply that I do not call those mental operations ‘diagrammatic reasoning’ (by contrast to many chapters in *Diagrammatic Reasoning: Cognitive and Computational Perspectives,* Glasgow et al. 1995). The function of diagrammatic reasoning, as I understand the term, is to explain how it is possible to develop those mental capacities by referring to concrete, visible activities that allow us, first of all, to develop ‘mental models.’ As we will see later, internal processes are necessary to explain the possibility of diagrammatic reasoning. Precisely for that reason I consider it an advantage to make a clear terminological distinction between *diagrammatic reasoning* as involving the construction of *external* diagrams and *mental modeling* that operates exclusively on mental models. Again, however, there might be a continuum between both these possibilities. When we sketch a diagram, for example, or indicate relations only by gesturing, the main point would be mental modeling that is supported by only vague external representations.

The third point regarding the function of diagrammatic reasoning as described above concerns the more fundamental question what a 'representation' is. I am following here Peirce's basic idea that semiotic relations are not dyadic relations—that is relations between signs and objects they represent—but *triadic* relations. A sign is only a sign when it is *interpreted* as a sign that signifies something. For Peirce, three elements constitute a semiotic relation: a sign, or representation; an object; and what he calls the 'interpretant' (Peirce CP 2.228). Since all three are only relevant as part of a triadic *relation*, there is no 'objective meaning' of signs. A sign's meaning depends on its interpretation. This intrinsic relativism of Peirce's semiotics is constrained both by a community of sign users and an evolutionary development of sign meanings that is integrated in the progress of science. By contrast to this social and scientific 'embeddedness' of a sign's *meaning*, Peirce's various distinctions of a huge amount of different *types* of signs—symbol, index, icon, diagram, type, token, etc.—are mainly constructed in a top-down approach, based on a complex set of epistemological and phenomenological considerations (Hoffmann 2005a). An implication of this top-down approach is that signs like a 'diagram' are only *formally* defined. Applying those definitions, however, to identify signs we are facing in concrete situations depends again on interpretation. 'Interpretation,' in this context, should be understood from a functional point of view as formulated in Peirce's 'pragmatic maxim': meanings become visible in the 'practical consequences' that result from a signs usage (Peirce, CP 5.9; CP 5.438).

Building on Peirce's definition of a diagram quoted above (CP 4.418), I would say that a *diagram* is an external representation of relations that is constructed according to the rules and conventions of, and by means of the elements and relations available in, a certain system of representation. Such a representational system provides the means, and constrains the possibilities, both of *constructing* diagrams and of any *manipulation* we perform on those diagrams. As already discussed above, this formal definition of a diagram does not exclude the possibility of calling also normal sentences or algebraic equations 'diagrams.' It is only due to the conventions of ordinary English that I consider diagrams here primarily as *spatial* representations of relations. It is a widely shared assumption that the function of diagrammatic reasoning described above can best be fulfilled with spatial, or graphical diagrams (Barwise & Etchemendy 1994; Craig et al. 2002).

For that reason, and based on several of Peirce's formulations that I quoted above, I define diagrammatic reasoning here in the form of a map (Figure 1). The representational system used to construct this diagram is Cmap, a freely available 'knowledge modeling kit' that has been developed by the Institute for Human and Machine Cognition (IHMC), Florida, as a concept mapping tool (http://cmap.ihmc.us/). The ontology provided by the software consists of a virtually

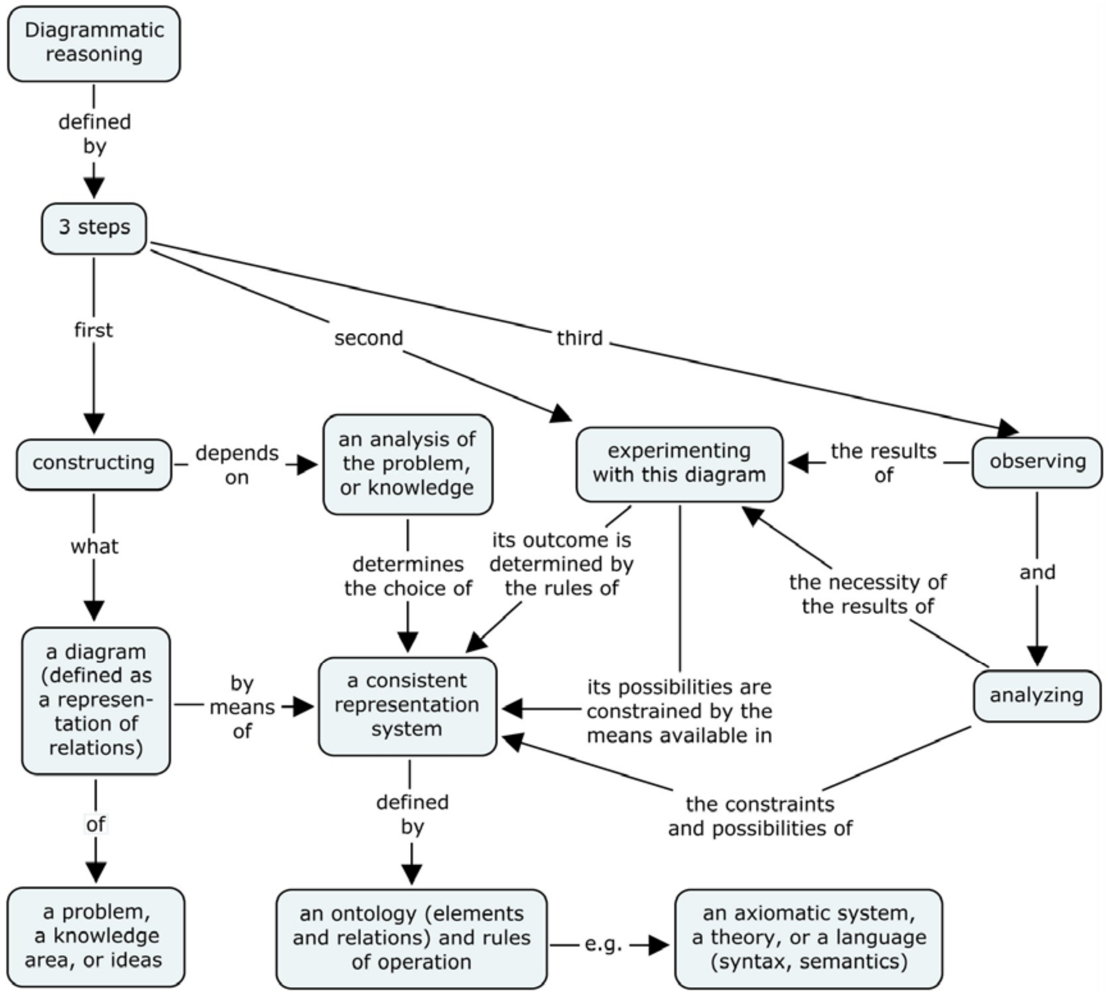

Figure 1: A diagrammatic definition of diagrammatic reasoning (created with IHMC Cmap tools: http://cmap.ihmc.us/)

infinite two-dimensional space, text boxes, and *n*-adic relations. The only rule that constrains the construction of diagrams here is that each relation has to be named. Thus, Cmap is a very flexible representation system that provides a lot of freedom. By defining additional rules and conventions, however, it can be used to develop more specific representational systems like the one I elaborated for Logical Argument Mapping (LAM; cf. Hoffmann 2005b).

This diagram facilitates an understanding of diagrammatic reasoning at least in so far as the central role of the chosen representation system becomes evident. No other text box gets more references from other text boxes than this one.

Elsewhere, I showed how based on this definition of diagrammatic reasoning a distinction of eight different forms of discovery and learning (abduction) can be developed (Hoffmann forthcoming-b). Here, however, my focus is different. The questions I want to answer in the second part of this paper are: What cognitive conditions must be fulfilled to learn something new by means of diagrammatic reasoning? What role do external representations play in cognitive processes?

2. Diagrams as elements of distributed cognition

Peirce was quite optimistic regarding the possibility of explaining learning processes and scientific discoveries—at least in mathematics—by diagrammatic reasoning:

> All our thinking is performed upon signs of some kind or other, either imagined or actually perceived. The best thinking, especially on mathematical subjects, is done by experimenting in the imagination upon a diagram or other scheme, and it facilitates the thought to have it before one's eyes. (Peirce NEM I 122)

From that he concluded that for any 'concept' or mental state 'external signs answer every purpose, and there is no need at all of considering what passes in one's mind' (ibid.). This assessment corresponds to the core idea of his semiotics that 'man is a sign' (CP 5.314). As Vincent Colapietro explains:

> In opposition to the dominant *mentalist* tradition that has defined signs as the expressions of minds, Peirce proposed a thoroughgoing semiotic perspective in which the reality of mind is seen as essentially the development of a system of signs. The mind is a species of semiosis. (Colapietro 1989 xx; cf. 97)

For Peirce, human reasoning is a *part* of the development of signs, not the other way around. This approach can be justified by the fact that the representational systems we are using have a reality independent of ourselves. It is beyond our power to define their rules and the meanings of their elements completely arbitrarily—they are the means of a culture, not of individuals. However, representational means are not simply 'out there,' they are *our* means to construct diagrams. In this way, representational means are private as well as public, external as well as internal, they are, at the same time, the means of a culture that we have at our disposal *and* means that live only in our thinking and acting.

Especially when it comes to learning and scientific discoveries we cannot simply claim that individuals are part of a universal semiosis. Discoveries are made by human beings. The challenge is, therefore, to explain the specific interaction that happens between an individual's cognitive processes and the culturally available representational means.

To meet this challenge, it would be nice to have a theory of cognition that could be used to clarify cognitive conditions of diagrammatic reasoning. Unfortunately, what cognitive science can offer at this point is hardly sufficient for my purposes. Understanding human cognition is still one of those tasks where we know more about problems and limitations of our approaches than about solutions. However, empirical research indicates at least that

- certain diagrams that are 'informationally' equivalent to sentential representations can nevertheless 'computationally' be more efficient when it comes to searching for information we need for problem solving, matching information to knowledge in long-term memory, and supporting 'perceptual inferences, which are extremely easy for humans' (Larkin & Simon 1987; critical Scaife & Rogers 1996: 195; but see also Healey et al. 2000 about the use of graphical media in synchronous communication. In their study, participants developed their use of graphics, producing progressively more abstract graphical representations as their experience increased);
- the cognitive support diagrammatic reasoning can provide depends heavily on the chosen system of representation. As J. J. Zhang & Norman (1994) show, carrying out a multiplication by means of roman numerals (e.g. LXVII × X) is much harder than doing the same task by arabic numbers (68 × 10). An important additional result of their study regarding different representational forms was that subjects were the more successful in problem-solving tasks the more rules of the representational system were directly accessible in the external representations themselves and had not been learnt independently;
- observable behavior—as has been shown with subjects playing Tic-Tac-Toe—is 'determined by the directly available information in external and internal representations in terms of perceptual and cognitive biases' in a way that Zhang (1997) suggests the idea of 'representational determinism': 'the form of a representation determines what information can be perceived, what processes can be activated, and what structures can be discovered from the specific representation';
- it is easier to work with already conventionalized representational systems than being forced to learn simultaneously domain knowledge and a novel representational system (Scaife & Rogers 1996: 206; Brna et al. 2001);
- the more advanced subjects' conceptual knowledge on a certain area is, the better they are at 'fine-tuning' diagrams according to the needs given by a task, at representational variability, and at using 'fine-tuned diagrams as tools "to think with" while reasoning' (Kindfield 1999);

- subjects performed better on certain tasks when they have either constructed diagrams in a preceding task that was structurally analogous to the target problem, or simulated a situation with wood blocks while studying an analogous problem (Craig et al. 2002);
- diagrammatic reasoning, while expected to reduce cognitive load, can itself increase this load so that learning possibilities are reduced instead of enlarged. Empirical research on how people work with computer supported argument visualization (CSAV), for instance, hints at the dilemma that 'the more specific an argument visualization technique is, the more it allows the users to disambiguate the problem and all of its aspects, the easier it will be to determine what the different perspectives on the problem are and the easier it should be to make sense of it and solve it. On the other hand, the more specific an argument the more complex it is to use it and the more room that is left open for not arguing the problem itself, but rather of discussing/arguing about the technique' (van Bruggen et al. 2003: 42). Similarly, Merrill & Reiser (1993) show that an 'additional working memory load' is imposed on students if the structure of a certain representational system does not match well 'with the structure of the students' plan.' (Cf. also Hegarty 2000);
- diagrammatic reasoning supports 'self-explanation' as 'an effective metacognitive strategy that can help learners develop deeper understanding of the material they study.' In an experiment by Ainsworth & Loizou (2003) subjects were 'presented with information about the human circulatory system and prompted to self-explain; 10 received this information in text and 10 in diagrams. Results showed that students given diagrams performed significantly better on post-tests than students given text. Diagrams students also generated significantly more self-explanations than text students. Furthermore, the benefits of self-explaining were much greater in the diagrams condition';
- there are limitations of using diagrammatic reasoning, and that 'diagrammatic thinking has to be substituted and complemented by conceptual-verbal reasoning,' although there are concepts whose 'intrinsic lack' of related diagrams 'poses what can be called an epistemological obstacle to learning that notion' (Dörfler 2005).

Although these are important empirical results, we are still facing what Mike Scaife and Yvone Rogers identified already in 1996 as the central problem regarding 'external cognition': 'we need insight into how people read and interact with diagrams' (206); we need an 'adequate cognitive processing model' (199). Scaife and Rogers criticize in particular the many approaches that commit what they call the 'resemblance fallacy': They cannot find any indication that the role diagrammatic reasoning plays for cognitive processes is based on a resemblance of 'external and internal representations,' or that graphic forms 'encourage students to create mental images that, in turn, make it easier for them to learn certain types of material' (199-201).

Especially if a structural equivalence is assumed between an external diagram and an internal 'mental model,' the cognitive problem posed by Scaife and Rogers is not resolved but simply duplicated. Every diagram is a *finite* and *distinct* entity formed by a set of countable, definite elements and relations; it is a more or less complex *sign* composed of symbols, indices, and icons. The idea of structural equivalence would imply that the corresponding 'mental model'—whatever its neurological or symbolic manifestation might be—must be representable as exactly the same finite and distinct sign. No sign, however, can by itself determine what it is referring to. Its meaning depends necessarily on *interpretation*. That means, any representation—be it an external diagram or an internal mental model—is only a representation if it is representing *something for somebody* (Peirce CP 2.228). Take for example a city map. In order to *use* the map, you need to know where you are on the map, and you must relate points on the map to points in the city. The map itself cannot establish any relation to what it represents. Without this activity and cognitive ability of a user which is *external* to the map itself, the map is without any use. The same is true with regard to any diagram or model that is supposed to represent something. Even if it contains indices whose function it is to direct your attention to elements outside of itself, you have to *interpret* these indices. If it contains symbols you have to know the conventionalized meanings of these symbols. Without interpretation—which can fail, of course, or be insufficient—a sign does not signify anything, as Peirce showed. Or take a mathematical proof. A proof is only a proof if it is *accepted* at least by some people as a proof. (This became clear with very complicated proofs like Andrew Wiles' proof of Fermat's 'Last Theorem,' or in the debate on whether computer proofs are acceptable). Signs are signs only in those functional settings in which they can be interpreted—or misinterpreted—by means of knowledge that is already given (Peirce CP 8.183; 8.178f.), but which can also be acquired and developed, of course. If all this is true, I cannot see how it could not be true for mental models as well, if these are conceived of as structurally equivalent to external diagrams.

Again, I do not deny the possibility of mental models or mental images. But the point is: such an assumption alone cannot explain how we interact with diagrams. We still need what has been called in good old-fashioned philosophy the '(epistemic) subject' (cf. Hutchins 1995, chap. 9, and the critique of 'encodism' in cognitive science formulated by Bickhard 1992, Bickhard 1996, et al.). We need something like a subject or self as that entity in a model of cognition that is responsible for *interpreting* representations based on interests, purposes, beliefs, values, emotions, and factors like his or her lifeworld, environment, and neurophysiological state—even if it turned out at the end that we have to conceptualize this 'subject' in a completely new, more interactive, social, and dynamic way (cf. Bickhard 2004; Maturana & Varela 1987; Mead 1913; Valsiner 2005).

To prepare at least some necessary steps with regard to cognitive conditions of diagrammatic reasoning, I will follow the philosophical method that was first developed by Immanuel Kant. Reacting to Hume's criticism at any attempt to justify knowledge by observation and inductive reasoning, Kant thought that the objectivity of knowledge can only be guaranteed by means of principles that must be given *before* any experience and observation. Of course, there are hardly any philosophers anymore who would accept those metaphysical principles, but the *strategy* Kant developed—he called it the 'transcendental method' (Kant CPR B 81)—can still be very productive. In a more general way, this strategy starts with assumptions that nobody could doubt seriously. Nobody would doubt, for example, that there is causality in our physical world. However, as Hume showed, causality can never be *observed* since we can never know whether two events—the only things that are observable—are connected by causality or simply by coincidence. At this point, says Kant, the only thing we can do is to take the assumed reality of causation as the starting point and ask ourselves which *conditions* must be fulfilled to explain the 'possibility' of those realities.

That is exactly what I will do in the following considerations. I take it for 'real' that we can learn by diagrammatic reasoning, and I ask what kind of cognitive *conditions* we have to presuppose to explain this possibility. By formulating those conditions, I hope to identify at least a set of *functions* a cognitive model should be able to describe.

A crucial advantage of such a transcendental, or functional, approach is that it starts from the *unity* of an individual's external and internal world. This way, there is no need to bridge the gap between the external and the internal that is inevitable if we separate both. I agree with Hutchins when he says that those cognitivist approaches that focus exclusively on internal processes are as wrong as those approaches that claim, like behaviorism or Peirce in the first two quotes of this section, 'that internal mental structure was either irrelevant or nonexistent' (371 f.). And I agree with him that a more promising strategy would be to expand the concept of a 'cognitive system' in a way that *external* elements—like the visible diagrams we are using in diagrammatic reasoning—are included. Diagrammatic reasoning is, first of all, a case of 'distributed cognition' as defined by Hutchins (1995; cf. Clark 1998; Clark & Chalmers 1998; Giere 2002; Hoffmann forthcoming-a). Cognitive processes are 'distributed' over a cognitive system that includes a diagram as something *external* to an individual's brain, but *internal* to the cognitive system.

To develop a better understanding of diagrammatic reasoning as a case of distributed cognition, and to clarify the question how mental, cognitive processes relate to, and are influenced by, diagrammatic reasoning, I will analyze an example.

My example can be taken as the oldest description of diagrammatic reasoning we know of, namely Plato's description of an exercise Socrates performs with an uneducated slave boy (Plato Men. 82b–86c). Although Plato's intention in this passage is quite the opposite of diagrammatic reasoning—his goal is to 'prove' his anamnesis thesis which claims that there is no learning, only a process of 'recollecting' what we saw before birth as the 'eternal form' of everything—it seems to be far more appropriate to interpret this first known lesson in mathematics as a demonstration of Peirce's claim that 'Diagrammatic reasoning is the only really fertile reasoning' (CP 4.571).

Socrates makes sure that the boy knows what a square is by drawing a figure like ABCD in Figure 2 in the sand (including something like the dotted lines) and asking him some questions about it. The side AB of this square is supposed to be two feet, and it turns out that the boy knows that the size of the square is thus four square feet, and that a square double the size of ABCD would be eight square feet (82c,d). After an agreement is reached about these knowledge items, Socrates asks the boy how long the *side* of the eight square feet square would be.

The boy's first suggestion is that the side should be 'twice the length' of the original square side, that is four feet long (82e). Obviously, he simply correlates doubling the area to doubling the side. But what does Socrates do to show that this answer is inadequate? He *experiments* with the diagram of the square he just drew and demonstrates the *implications* of the boy's suggestion at the concrete figure. This operation results in the square AGFE (Figure 2). Looking at this big square, the boy must admit that his answer yields a square that is four times the size, and not twice, of the original square (83b).

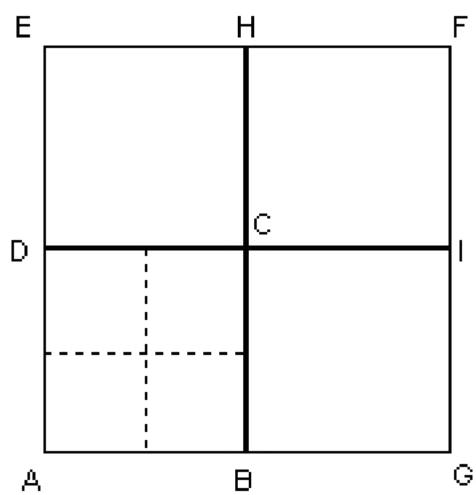

Figure 2: What is the side length of a square double the size of ABCD?

Before we continue with Plato's story, let us consider what kind of cognitive processes are involved so far. One point gets highlighted by Socrates immediately. The obvious disappointment the boy experiences regarding his expectations is an essential step in his learning process. Since he is able to acknowledge his ignorance regarding the matter, he is 'in a better position' now. In this new situation 'he would be glad' to find out the right answer, 'whereas before he thought he could easily make many fine speeches to large audiences about the square of double size and said that it must have a base twice as long' (84b,c). Seeing the necessary implications of his premature assumption *motivates* the boy to search for the correct solution. Socrates' experiment with the diagram is a way to produce this motivation.

However, what is it in this process of experimenting with the diagram that *forces* the boy to give up his first hypothesis? Although it might sound trivial from an educated point of view, it is crucial to note here that the boy's insight in his failure is only possible if he fulfills the following cognitive conditions. He has to accept, first, that doubling the side of ABCD leads necessarily to AGFE, second, that there is a contradiction between this result and his expectation and, third, that contradictions are not acceptable, they have to be resolved. (As Lewis Carroll 1895 taught us in his funny story 'What the Tortoise Said to Achilles,' it is quite possible that someone does not accept logical necessity, even if it is hard to talk to those people).

The first one of these conditions depends obviously on what is essential for diagrammatic reasoning: the consistency and rationality of the representation system we choose to construct a diagram. As noted in my definition of diagrammatic reasoning above (Figure 1), the outcome of any experiment we perform with a diagram is *determined* by the rules and conventions of the respective representational system. Observing the experiment with the original square, nobody would doubt that doubling the side of ABCD leads to AGFE, but this evidential truth is grounded in two very different things: on the one hand in the *objective* truth that a consistent representational system like Euclidean geometry determines necessarily the outcome of the observable operation on the diagram and, on the other, in the *subjective* precondition that we are able to *accept* this objective truth. (By contrast to the usual epistemological considerations, 'objectivity' itself is not a problem in this case since the representational system that guarantees objectivity has deliberately been created for this very purpose: to guarantee objectivity for geometrical operations). It is essential that, on the one hand, the representation systems we are choosing to construct a diagram are *cultural* tools whose validity, usefulness, or appropriateness is *socially* established so that individuals cannot change it by their own choice and, on the other, that the outcome of any experiment with such a diagram is necessary and true *for us*.

Based on this, we can say the boy's first step of learning by means of diagrammatic reasoning depends on the following cognitive conditions. He has to

1. know the rules and conventions of a representational system that determine the construction of a diagram and the outcome of experiments with it (at least so far as they are relevant for a concrete problem);
2. accept these rules and conventions;
3. accept the principle of non-contradiction;
4. feel forced to avoid contradictions, and to be motivated to look for a resolution of contradictions.

If we don't accept these four points as preconditions, we could not explain the boy's insight into his failure, and his motivation to do it better. But this assumption leads to some serious and interesting problems. So far, we followed a transcendental strategy, that is we started from the fact that the boy realizes his failure and we looked for the conditions that have to be fulfilled to provide this possibility. At this point, a Kantian transcendentalist would argue that something like the principle of non-contradiction is part of our 'a priori knowledge,' that is we have to presuppose it as given before any experience. However, we know that small children do not have any problems with accepting contradictions—they simply do not care. And the rules and conventions of Euclidean geometry—as its name tells—were formulated by Euclid. Neither did they exist before Euclid formulated them, nor are they necessary as we know from non-Euclidean geometries. Therefore, it seems to be more appropriate to replace any a priori assumptions at this point by the idea that whatever we have to presuppose as the boy's cognitive preconditions is itself the result of *development* and learning.

This consideration, however, leads to the first of the problems I mentioned above. If we assume that the cognitive conditions that are necessary for diagrammatic reasoning have to be developed themselves, how do we develop them? Although the boy in Plato's example seems to fulfill all these conditions, it is not hard to imagine what would happen if he would fail to do so. Obviously, Socrates would teach him what he needs to know to perform the process of diagrammatic reasoning. This would be easy in this case regarding the rules of the used representational system, and it wouldn't be much harder to teach him the principle of non-contradiction. However, we should keep in mind two important points: on the one hand, that the four points listed above are nevertheless absolutely basic cognitive abilities and, on the other, that they have to be accepted without any exception. These are *general* rules of Euclidean geometry and *general* logical principles; we are neither allowed to change the rules and conventions of a representation system during diagrammatic reasoning, nor to switch between accepting and rejecting the principle of non-contradiction. If we would commit one of those 'crimes'—and young children do this pretty often—there would be considerable *social intervention* to make us accepting them.

Mark Bickhard (1992) highlights in his 'interactivist' approach to cognition—building on evolutionary and genetic epistemology (Campbell 1974; Piaget 1970)—the influence of 'selection pressure' as a condition of developing those procedures and representational contents that are adequate in a certain type of situation regarding a certain goal. We only keep as cognitive tools what helped us to *accomplish* a certain goal. If you want to satisfy your hunger, you better learn your motor control in a way that you can handle a spoon if only mash is available. And what seems to work with infants based on pressure provided by our physical environment, seems to work as well when it comes to select those logical principles and rules of representational systems that we need to perform diagrammatic reasoning. In this case, however, it is the *social* environment—parents, teachers, and peers—that tells us what is acceptable and what is not. Teaching and social pressure seem to be the primary factors in developing those conditions of diagrammatic reasoning that are necessary to perform this reasoning.

Learning by social interaction, however, is itself only possible if we presuppose an even 'deeper' level of cognitive abilities. In order to learn by social interventions we have to be able to understand what people are telling us, we must be able to make sense of their words in relation to a given situation, and we must be willing to follow their suggestions—an ability that seems to decrease when people become older—and so on. Thus, the solution of our first problem concerning the development of those cognitive conditions we have to presuppose to explain the boy's disappointment leads us to the assumption of a complex, hierarchical order of cognitive conditions. This order may vary from situation to situation.

A second problem might be more serious. If the boy fulfills the four conditions listed above, why does he come up with his wrong answer in the first place? Knowing the rules of Euclidean geometry and accepting the principle of non-contradiction obviously *excludes* the possibility that the side of the doubled square is four feet long. Why, then, does he suggest this? This question can only be answered if we are more careful with regard to the meaning of 'knowing' Euclidean geometry and the principle of non-contradiction. Obviously, the boy does not 'know' these things in a way that this knowledge would prevent him from suggesting a wrong answer to Socrates' question. But what kind of knowledge does he have?

His answer to Socrates' question in which he suggests that the double-sized square should have a side 'twice the length' of the original square assumes that he only has confused ideas about the relations between area sizes and lengths. In this situation, the central role of diagrammatic reasoning becomes evident. There would be no need to construct a diagram if the boy were already able to give the correct answer. The function of diagrammatic reasoning is—as noted above—to *facilitate thinking processes in situations that are too complex to be coped with exclusively by internal cognitive means*. For the boy, the problem of determining the side length of a doubled square is too complicated. He needs to observe Socrates' manipulation of the diagram *to see* what is going on if he doubles the side. The process of diagrammatization allows him to clarify his vague ideas and to coordinate what he already knows in a way that makes clear to him that his first answer was wrong.

This function of diagrammatic reasoning in the boy's learning process can be described by the metaphor of 'scaffolding' which has been introduced to psychological research first by Wood et al. (1976) in a discussion that goes back to Vygotsky's idea of the 'zone of proximal development' (cf. Rogoff & Wertsch 1984). While they used the term to characterize the role of a more knowledgeable individual for a learner—a teacher or parent who provides more advanced knowledge that works like a temporary framework used in the construction of buildings—more recent talk on 'scaffolding' is broader (cf. Sherin et al. 2004; Renninger & Granott 2005). It includes any sort of external support 'that makes a particular learning process possible and that can be discarded after the learning has taken place' (van Geert & Steenbeek 2005: 116). However, the more it becomes clear that scaffolding is a useful concept to describe the possibility of learning, the more difficult it becomes to identify an understanding of this process, and a definition of the concept, upon which a majority of scholars would agree (cf. Sherin et al. 2004). Additionally,

> While 'scaffolding' is intuitively a compelling description of the process that can occur during interaction, details about how and why scaffolding works as it does are still being compiled. Its measurement is complex. (Renninger & Granott 2005: 111)

In this situation, it might be more adequate to continue the analysis of our example in order to get a better understanding of what 'scaffolding' could mean in this concrete case. The first point I would emphasize is that in Socrates' interaction with the boy we find a combination of 'scaffolding by persons' and 'scaffolding by means of a diagram.' Obviously, Socrates' success depends heavily on the fact that he draws a diagram representing the problem. However, we can imagine a situation in which the boy himself works with a diagram without an 'expert' helping him. In this case, we could talk about 'self-scaffolding' (cf. Bickhard 1992; Bickhard 2005; Granott 2005; Mascolo 2005). Thus, the diagram can be interpreted as a 'scaffold' that the learner constructs to help himself in a situation that is too complex for him to cope with.

Be it self-scaffolding or social scaffolding as in the *Meno*, the cognitive function the diagram fulfills in our example is based on the ten characteristics of diagrammatic reasoning I listed in the first section of this paper. Especially the clarification and coordination of confused ideas about the problem in question is crucial. If the boy in Plato's example had clear knowledge of geometry, he could answer Socrates' question without using a diagram. That means—coming back to our second problem—that we should clearly distinguish between *knowing* the four conditions of being disappointed listed above and something weaker than 'knowing.' Elsewhere, I suggested a distinction between 'knowledge' and 'cognitive abilities' to analyze more precisely what is going on in learning processes like the one described in the *Meno* (Hoffmann forthcoming-a). While 'knowing something' can be defined as being able to perform a certain activity—including the activity of formulating a proposition, or an argument—without being dependent on something which is given in the respective environment, a 'cognitive ability' would be an ability which is dependent on something else. This way, we could say that though the boy does not have *knowledge* of the rules and conventions of geometry and the principle of non-contradiction, he has nevertheless the *cognitive ability* to clarify those rules, conventions, and principles *by means of the diagram* Socrates draws. The diagram, therefore, is an essential part of the boy's cognitive system since—by definition—there is no cognitive ability without a corresponding object.

Based on this terminological distinction we can say that the main goal of learning is the transformation of cognitive abilities into knowledge; the transformation of activities that are dependent on external support into those that are independent and 'abstract,' so to speak. In our example, working with a diagram is crucial for this transformation. The process of diagrammatic reasoning is a process of disambiguation, clarification, and coordination of vague ideas.

Let's turn again to Plato's text. The question regarding the side length of the doubled square is still unanswered. Motivated by Socrates' pretty misleading hint that the 'line on which the eight-foot square is based must then be longer than this one of two feet [AD in Figure 2], and shorter than that one of four feet [AE in Figure 2]' (83d), the boy's second suggestion is that it must be three feet long, which is exactly the middle between two and four feet. This time, Socrates uses the diagram only to show that 'the whole figure' of a square with a side length of three feet will be 'three times three feet.' Based on the boy's *knowledge* that 'three times three feet' is nine feet, he realizes quickly that his second answer also was wrong (83e).

Now, how can we solve the problem? Plato presents Socrates as a real master of a scaffolding technique, at least initially. By counting piece by piece the four equally sized squares that form as a whole the big square AGFE in Figure 2, he makes evident that 'the whole figure' is four times larger than the original square (84d,e). His emphasis on the fact that we get four squares when we need the size of two could have been a great scaffold to induce the idea that we only need to divide each of these four squares by its diagonal to produce the correct solution (cf. Figure 3). But Socrates has to prompt the boy again to get him on the right track:

SOCRATES: Well then, how many times is the whole figure larger than this one?—Four times.

SOCRATES: But we should have had one that was twice as large, or do you not remember?—I certainly do.

SOCRATES: Does not this line from one corner to the other cut each of these figures in two? [He hints at BD, DH, HI, and IB in Figure 3]—Yes.

SOCRATES: So these are four equal lines which enclose this figure? [BDHI]—They are.

SOCRATES: Consider now: how large is the figure?—I do not understand.

SOCRATES: Within these four figures, each line cuts off half of each, does it not?—Yes.

SOCRATES: How many of this size are these in the figure? [Triangles of the size of DBC in BDHI]—Four.

SOCRATES: How many in this? [Triangles in ABCD]—Two.

SOCRATES: What is the relation of four to two?—Double.

SOCRATES: How many feet in this? [Square feet in BDHI]—Eight.

SOCRATES: Based on what line?—This one [DB].

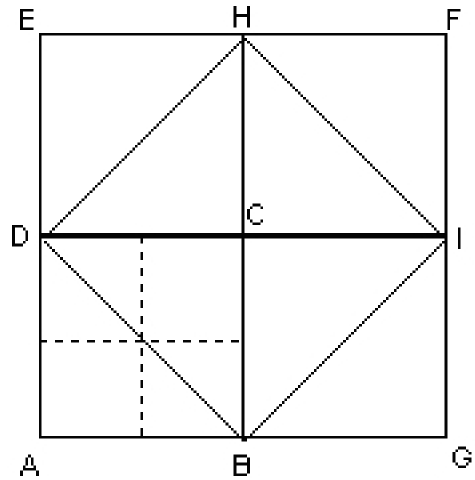

Figure 3: Doubling the square ABCD

SOCRATES: That is, on the line that stretches from corner to corner of the four-foot figure?—Yes.—Clever men call this the diagonal, so that if diagonal is its name, you say that the double figure would be that based on the diagonal?—Most certainly, Socrates. (Plato Men., 84e-85b)

Obviously, the last lines of this dialogue are not really a good example for sensible scaffolding. However, answering the question by looking at the diagonals in the 16 square feet figure is a genuine *creative* act. We could say that Socrates *prepares* this last step perfectly by constructing, first of all, Figure 2 and by emphasizing the fact that in this big figure we get four squares of the same size as the original one. But *using* this information in a creative way to solve the problem is something special.

From the boy's point of view, there are no further cognitive abilities that he needs in order to understand the correctness of Socrates' solution beyond what we have discussed already. Based on what he knows about Euclidean geometry it is evident that Socrates' approach solves the problem. The interesting question is, however, how it could be possible for him to find this answer without a tutor helping him. Let us assume he used a diagram like Figure 2 in a process of self-scaffolding. Based on what kind of cognitive conditions could it be possible for him to find the correct solution in a genuine *creative* act?

Peirce offers a set of terminological suggestions that can be used here to describe activities that have both a methodological and a cognitive dimension (cf. Hoffmann 2005a, 2005c, forthcoming-b). His concept of a 'theoric transformation' would be useful to describe the change of perspective that is necessary to *see* already in Figure 2 the possibility of the diamond-shaped square of Figure 3, and his notion of 'theorematic deduction' might be helpful to name the process of performing 'an ingenious experiment upon the diagram, and by the observation of the diagram, so modified, ascertains the truth of the conclusion' (Peirce CP 2.267; cf. CP 7.204). For

Peirce, such an ‘ingenious experiment’ can be realized by introducing ‘auxiliary individuals into the argument’ like the diagonal in Figure 3 (Hintikka 1983 <1980>, 113, cf. 109f.; Peirce EP II 96). However, all these terminological suggestions in themselves do not explain how it might be possible for the boy to perform something like a theoric transformation, or a theorematic deduction, in a way that the problem can be solved.

Analyzing a historical example for a theoric transformation in projective geometry, Peirce emphasizes that the genuine creative step was possible in this case for somebody who was already ‘acquainted’ with looking at diagrams from a certain point of view (Peirce SEM III 310 f.; cf. Hoffmann 2005a: 206-214; for empirical evidence see Craig et al. 2002). Similarly, if somebody has already studied the role of diagonals in squares, this person would be best prepared to associate her or his knowledge with the problem in question. At this point, it makes sense to talk about mental models as a precondition of diagrammatic reasoning. But these are not internal replicas of the external diagrams we are constructing in diagrammatic reasoning, but certain units of structured experience we can use to develop certain *perspectives on* those diagrams. This way, a mental model would be a cognitive means to *frame* the perception of a diagram—or, generally, of a problem—in a certain way. Playing with those model-based perspectives might be the central cognitive condition of successful diagrammatic reasoning.

3. Conclusion

If we look at diagrammatic reasoning as a process in which an individual (or a group of individuals) constructs an external representation, and experiments with this representation playfully and creatively, in order to clarify, structure, and coordinate thinking processes, we can summarize our considerations regarding cognitive conditions of diagrammatic reasoning as follows. Most important might be that in diagrammatic reasoning we are facing an interplay between an individual’s *internal* cognitive processes and the *objective* rules and conventions of a representation system she or he chooses to construct a diagram. These rules are not—at least not during the process of diagrammatic reasoning itself—at the individual’s disposal; they are anchored in the general ways a certain community or culture uses this system of representation. This externality, or objectivity, of the representation system is crucial for the possibility of learning by means of diagrammatic reasoning. Only if we accept the objectivity and constraining power of these rules and conventions can we be challenged by what turns out as necessary implications of an experiment we perform with our diagram. Only if the boy in Plato’s *Meno* accepts what Socrates shows at his diagram can he see the disillusioning contradiction between his expectations and this outcome. It is the rationality implemented in Euclidean geometry itself that makes that a diagram, as Kathleen Hull (1994) puts it, ‘becomes the something (non-ego) that stands up against our consciousness’; ‘reasoning unfolds when we inhibit the active side of our consciousness and allow things to act on us’ (287).

My analysis of Plato’s example of how to double a square illuminated in particular two theses: first, that the cognitive ability necessary to perform diagrammatic reasoning, and to develop creative problem solutions, can be reconstructed as a more or less complex hierarchy of further cognitive abilities and forms of knowledge (which could partly be reconstructed as mental models) that includes accepting the principle of non-contradiction, the ability to listen to, and to understand, experts, knowledge of Euclidean geometry, experience with the role of diagonals in squares, and so on (cf. Hoffmann & Roth 2004); second, that the role of diagrammatic reasoning can only be understood adequately if we conceive it as part of cognitive processes whose central feature is the clarification, disambiguation, and coordination of confused and vague ideas which must be given already. This way, diagrammatic reasoning takes place in those forms of cognition whose essence can best be captured by the concept of ‘distributed cognition.’

Acknowledgements

* Many thanks to my colleagues and friends Bryan Norton, Nancy Nersessian, Jan Schmidt, Roberta Berry, and Robert Kirkman for a stimulating discussion of an earlier version of this paper.

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